On a Higher-Order Systems of Difference Equations

[1,2]E. M. Elsayed*, [1,3]Marwa M. Alzubaidi

[1]King Abdulaziz University, Faculty of Science, Mathematics Department, P.O. Box 80203, Jeddah 21589, Saudi Arabia

[2]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

[3]Mathematics Department, The university College of Duba, University of Tabuk, Tabuk, Saudi Arabia

*Corresponding author: emmelsayed@yahoo.com

Abstract

Our goal in this objective is to study the form of the solutions of a class of rational systems of difference equations:

$$x_{n+1} = \frac{y_{n-5}x_{n-8}}{y_{n-2}\left(1 + y_{n-5}x_{n-8}\right)},\ y_{n+1} = \frac{x_{n-5}y_{n-8}}{x_{n-2}(\pm 1 \pm x_{n-5}y_{n-8})},\quad n = 0, 1, \ldots,$$

where the initial conditions $x_{-\alpha}$, $y_{-\alpha}$, $\alpha \in \{0, 1, \ldots, 8\}$ are non-zero real numbers.

Keywords: Difference equations, Systems of difference equations, Recursive sequences.

1 Introduction

Difference equations has very important in the construction of mathematical models which have been used by researchers from other fields such as biology (population dynamics in particular), ecology, engineering and economics, to give simplified solve of real-life problems. The studies in this interesting area of research will continue to emerge and evolve. The issues of stability and attractivity in nonlinear difference equations constitute an essential part in the contributions that have been made to the theory of difference equations. For more results about global character and local asymptotic stability, see, for instance, [1-56] and the references cited therein.

Ahmed and Elsayed [1] have got the expressions of solutions of some rational difference equations systems

$$x_{n+1} = \frac{x_{n-1}y_{n-2}}{y_n\left(-1 \pm x_{n-1}y_{n-2}\right)}, \quad y_{n+1} = \frac{y_{n-1}x_{n-2}}{x_n\left(\pm 1 \pm y_{n-1}x_{n-2}\right)}.$$

Dekkar et al. [4] obtained the global stability of a third-order nonlinear system of difference equations with period-two coefficients

$$x_{n+1} = \frac{p_n + y_n}{p_n + y_{n-2}}, \quad y_{n+1} = \frac{q_n + x_n}{q_n + x_{n-2}}.$$

In [7] Din investigated the boundedness character, the local asymptotic stability of equilibrium points and global of the unique positive equilibrium point of a discrete perdator-pery model given by

$$x_{n+1} = \frac{\alpha x_n - \beta x_n y_n}{1 + \gamma x_n}, \quad y_{n+1} = \frac{\delta x_n y_n}{x_n + \eta y_n}.$$

El-Dessoky [9] obtained the solution for rational systems of difference equations of order three

$$x_{n+1} = \frac{y_{n-2}}{-1 \pm y_{n-2}x_{n-1}y_n}, \qquad y_{n+1} = \frac{x_{n-2}}{\pm 1 \pm x_{n-2}y_{n-1}x_n}.$$

In [10] El-Dessoky and Elsayed studied the solution and periodic nature of some systems of rational difference equations

$$x_{n+1} = \frac{x_n y_{n-1}}{y_{n-1} \pm y_n}, \qquad y_{n+1} = \frac{y_n x_{n-1}}{x_{n-1} \pm x_n}.$$

El-Dessoky et al. [11] obtained the solutions of the following rational systems of difference equations

$$x_{n+1} = \frac{x_{n-3}y_{n-4}}{y_n\left(\pm 1 \pm x_{n-3}y_{n-4}\right)}, \quad y_{n+1} = \frac{y_{n-3}x_{n-4}}{x_n\left(\pm 1 \pm y_{n-3}x_{n-4}\right)}.$$

Elsayed and Ibrahim [22] solved solutions for some systems of nonlinear rational difference equations

$$x_{n+1} = \frac{x_{n-2}y_{n-1}}{y_n(\pm 1 \pm x_{n-2}y_{n-1})}, \qquad y_{n+1} = \frac{y_{n-2}x_{n-1}}{x_n(\pm 1 \pm y_{n-2}x_{n-1})}.$$

Elsayed and Alghamdi [23] solved the form of the solution of nonlinear difference equation systems

$$x_{n+1} = \frac{x_{n-7}}{1 + x_{n-7}y_{n-3}}, \quad y_{n+1} = \frac{y_{n-7}}{\pm 1 \pm y_{n-7}x_{n-3}}.$$

Haddad et al. [32] obtained solution form of a higher-order system of difference equations and dynamical behavior of its special case

$$x_{n+1} = \frac{x^p_{n-k+1}y_n}{ay^p_{n-k} + by_n}, \qquad y_{n+1} = \frac{y^p_{n-k+1}x_n}{\alpha x^p_{n-k} + \beta x_n}.$$

In [44] Kurbanli studied the behavior of solutions of the following system of difference equations

$$x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}-1},\quad y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}-1}.$$

Kurbanli et al. [45], [46] obtained the solutions of following systems

$$x_{n+1}=\frac{x_{n-1}+y_n}{y_nx_{n-1}-1},\quad y_{n+1}=\frac{y_{n-1}+x_n}{x_ny_{n-1}-1}.$$
$$x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}+1},\quad y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}+1}.$$

Mansour et al. [47] investigated the solutions and periodicity of some systems of difference equations

$$x_{n+1}=\frac{x_{n-5}}{-1+x_{n-5}y_{n-2}},\quad y_{n+1}=\frac{y_{n-5}}{\pm1\pm y_{n-5}x_{n-2}}.$$

Our goal in this article is to investigate the solutions of the following nonlinear difference equations systems

$$x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}\left(1+y_{n-5}x_{n-8}\right)},\quad y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}\left(\pm1\pm x_{n-5}y_{n-8}\right)},\quad n=0,1,...,$$

where the initial conditions are non-zero real numbers.

2 Main Results

2.1 The First System $x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}(1+y_{n-5}x_{n-8})}$, $y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}(1+x_{n-5}y_{n-8})}$

In this section, we get the solutions of the system of higher order difference equations in the form

$$x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}\left(1+y_{n-5}x_{n-8}\right)},\ \ y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}\left(1+x_{n-5}y_{n-8}\right)},\tag{1}$$

with nonzero real initial conditions $x_{-\alpha}, y_{-\alpha}$, $\alpha\in\{0,1,2,...8\}$.

Theorem 1. *If* $\{x_n, y_n\}$ are solutions of difference equation system (1). Then For $n=0,1,2,...$

$$x_{12n-8}=\frac{(gt)^n}{a^{n-1}l^n}\prod_{i=0}^{n-1}\frac{(1+(2i+1)dl)\,(1+2iaq)}{(1+2igq)\,1+(2i+1)dt},$$
$$x_{12n-7}=\frac{(hu)^n}{b^{n-1}m^n}\prod_{i=0}^{n-1}\frac{(1+(2i+1)em)\,(1+2ibr)}{(1+2ihr)\,1+(2i+1)eu},$$

$$x_{12n-6} = \frac{(kv)^n}{c^{n-1}p^n}\prod_{i=0}^{n-1}\frac{(1+(2i+1)fp)\,(1+2ics)}{(1+2iks)\,1+(2i+1)fv)},$$

$$x_{12n-5} = d\left(\frac{al}{gt}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)gq)\,(1+2idt)}{(1+(2i+2)dl)\,1+(2i+1)aq)},$$

$$x_{12n-4} = e\left(\frac{bm}{hu}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)hr)\,(1+2ieu)}{(1+(2i+2)em)\,1+(2i+1)br)},$$

$$x_{12n-3} = f\left(\frac{cp}{kv}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)ks)\,(1+2ifv)}{(1+(2i+2)fp)\,1+(2i+1)cs)},$$

$$x_{12n-2} = g^{n+1}\left(\frac{t}{al}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)dl)\,(1+(2i+2)aq)}{(1+(2i+2)gq)\,1+(2i+1)dt)},$$

$$x_{12n-1} = h^{n+1}\left(\frac{u}{bm}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)em)\,(1+(2i+2)br)}{(1+(2i+2)hr)\,1+(2i+1)eu)},$$

$$x_{12n} = k^{n+1}\left(\frac{v}{cp}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)fp)\,(1+(2i+2)cs)}{(1+(2i+2)ks)\,1+(2i+1)fv)},$$

$$x_{12n+1} = \frac{q}{1+aq}\left(\frac{l}{g}\right)^n\left(\frac{a}{t}\right)^{n+1}\prod_{i=0}^{n-1}\frac{(1+(2i+1)gq)\,(1+(2i+2)dt)}{(1+(2i+2)dl)\,1+(2i+1)aq)},$$

$$x_{12n+2} = \frac{r}{1+br}\left(\frac{m}{h}\right)^n\left(\frac{b}{u}\right)^{n+1}\prod_{i=0}^{n-1}\frac{(1+(2i+1)hr)\,(1+(2i+2)eu)}{(1+(2i+2)em)\,1+(2i+1)br)},$$

$$x_{12n+3} = \frac{s}{1+cs}\left(\frac{p}{k}\right)^n\left(\frac{c}{v}\right)^{n+1}\prod_{i=0}^{n-1}\frac{(1+(2i+1)ks)\,(1+(2i+2)fv)}{(1+(2i+2)fp)\,1+(2i+1)cs)},$$

$$y_{12n-8} = \frac{(gt)^n}{l^{n-1}a^n}\prod_{i=0}^{n-1}\frac{(1+(2i+1)aq)\,(1+2idl)}{(1+(2i+1)gq)\,(1+2idt)},$$

$$y_{12n-7} = \frac{(hu)^n}{m^{n-1}b^n}\prod_{i=0}^{n-1}\frac{(1+(2i+1)br)\,(1+2iem)}{(1+(2i+1)hr)\,(1+2ieu)},$$

$$y_{12n-6} = \frac{(kv)^n}{p^{n-1}c^n}\prod_{i=0}^{n-1}\frac{(1+(2i+1)fp)\,(1+(2i+1)cs)}{(1+(2i+1)ks)\,1+2ifv)},$$

$$y_{12n-5} = q\left(\frac{al}{gt}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)dt)\,(1+2igq)}{(1+(2i+1)dl)\,1+(2i+2)aq)},$$

$$y_{12n-4} = r\left(\frac{bm}{hu}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)eu)\,(1+2ihr)}{(1+(2i+1)em)\,1+(2i+2)br)},$$

$$y_{12n-3} = s\left(\frac{cp}{kv}\right)^n\prod_{i=0}^{n-1}\frac{(1+(2i+1)fv)\,(1+2iks)}{(1+(2i+1)fp)\,1+(2i+2)cs)},$$

$$y_{12n-2} = t^{n+1}\left(\frac{g}{al}\right)^{n}\prod_{i=0}^{n-1}\frac{(1+(2i+2)dl)\,(1+(2i+1)aq)}{(1+(2i+1)gq)\,1+(2i+2)dt)},$$

$$y_{12n-1} = u^{n+1}\left(\frac{h}{bm}\right)^{n}\prod_{i=0}^{n-1}\frac{(1+(2i+2)em)\,(1+(2i+1)br)}{(1+(2i+1)hr)\,1+(2i+2)eu)},$$

$$y_{12n} = v^{n+1}\left(\frac{k}{cp}\right)^{n}\prod_{i=0}^{n-1}\frac{(1+(2i+2)fp)\,(1+(2i+1)cs)}{(1+(2i+1)ks)\,1+(2i+2)fv)},$$

$$y_{12n+1} = \frac{d}{1+dl}\left(\frac{a}{t}\right)^{n}\left(\frac{l}{g}\right)^{n+1}\prod_{i=0}^{n-1}\frac{(1+(2i+1)dt)\,(1+(2i+2)gq)}{(1+(2i+2)aq)\,1+(2i+3)dl)},$$

$$y_{12n+2} = \frac{e}{1+em}\left(\frac{b}{u}\right)^{n}\left(\frac{m}{h}\right)^{n+1}\prod_{i=0}^{n-1}\frac{(1+(2i+1)eu)\,(1+(2i+2)hr)}{(1+(2i+2)br)\,1+(2i+1)em)},$$

$$y_{12n+3} = \frac{f}{1+fp}\left(\frac{c}{v}\right)^{n}\left(\frac{p}{k}\right)^{n+1}\prod_{i=0}^{n-1}\frac{(1+(2i+1)fv)\,(1+(2i+2)ks)}{(1+(2i+2)cs)\,1+(2i+1)fp)},$$

where $x_{-8} = a$, $x_{-7} = b$, $x_{-6} = c$, $x_{-5} = d$, $x_{-4} = e$, $x_{-3} = f$, $x_{-2} = g$, $x_{-1} = h$, $x_0 = k$, $y_{-8} = l$, $y_{-7} = m$, $y_{-6} = p$, $y_{-5} = q$, $y_{-4} = r$, $y_{-3} = s$, $y_{-2} = t$, $y_{-1} = u$ and $y_0 = v$.

Proof. For $n = 0$ the result holds. Hence assume that $n > 0$ and that our assumption hold for $n-1$, that is,

$$x_{12n-20} = \frac{(gt)^{n-1}}{a^{n-2}l^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(2i+1)dl)\,(1+2iaq)}{(1+2igq)\,1+(2i+1)dt},$$

$$x_{12n-19} = \frac{(hu)^{n-1}}{b^{n-2}m^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(2i+1)em)\,(1+2ibr)}{(1+2ihr)\,1+(2i+1)eu)},$$

$$x_{12n-18} = \frac{(kv)^{n-1}}{c^{n-2}p^{n-1}}\prod_{i=0}^{n-2}\frac{(1+(2i+1)fp)\,(1+2ics)}{(1+2iks)\,1+(2i+1)fv)},$$

$$x_{12n-17} = d\left(\frac{al}{gt}\right)^{n-1}\prod_{i=0}^{n-2}\frac{(1+(2i+1)gq)\,(1+2idt)}{(1+(2i+2)dl)\,1+(2i+1)aq)},$$

$$x_{12n-16} = e\left(\frac{bm}{hu}\right)^{n-1}\prod_{i=0}^{n-2}\frac{(1+(2i+1)hr)\,(1+2ieu)}{(1+(2i+2)em)\,1+(2i+1)br)},$$

$$x_{12n-15} = f\left(\frac{cp}{kv}\right)^{n-1}\prod_{i=0}^{n-2}\frac{(1+(2i+1)ks)\,(1+2ifv)}{(1+(2i+2)fp)\,1+(2i+1)cs)},$$

$$x_{12n-14} = g^n \left(\frac{t}{al}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)dl)\,(1 + (2i + 2)aq)}{(1 + (2i + 2)gq)\,1 + (2i + 1)dt)},$$

$$x_{12n-13} = h^n \left(\frac{u}{bm}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)em)\,(1 + (2i + 2)br)}{(1 + (2i + 2)hr)\,1 + (2i + 1)eu)},$$

$$x_{12n-12} = k^n \left(\frac{v}{cp}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)fp)\,(1 + (2i + 2)cs)}{(1 + (2i + 2)ks)\,1 + (2i + 1)fv)},$$

$$x_{12n-11} = \frac{q}{1 + aq} \left(\frac{l}{g}\right)^{n-1} \left(\frac{a}{t}\right)^n \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)gq)\,(1 + (2i + 2)dt)}{(1 + (2i + 2)dl)\,1 + (2i + 1)aq)},$$

$$x_{12n-10} = \frac{r}{1 + br} \left(\frac{m}{h}\right)^{n-1} \left(\frac{b}{u}\right)^n \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)hr)\,(1 + (2i + 2)eu)}{(1 + (2i + 2)em)\,1 + (2i + 1)br)},$$

$$x_{12n-9} = \frac{s}{1 + cs} \left(\frac{p}{k}\right)^{n-1} \left(\frac{c}{v}\right)^n \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)ks)\,(1 + (2i + 2)fv)}{(1 + (2i + 2)fp)\,1 + (2i + 1)cs)},$$

$$y_{12n-20} = \frac{(gt)^{n-1}}{l^{n-2}a^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)aq)\,(1 + 2idl)}{(1 + (2i + 1)gq)\,(1 + 2idt)},$$

$$y_{12n-19} = \frac{(hu)^{n-1}}{m^{n-2}b^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)br)\,(1 + 2iem)}{(1 + (2i + 1)hr)\,(1 + 2ieu)},$$

$$y_{12n-18} = \frac{(kv)^{n-1}}{p^{n-2}c^{n-1}} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)fp)\,(1 + (2i + 1)cs)}{(1 + (2i + 1)ks)\,1 + 2ifv)},$$

$$y_{12n-17} = q \left(\frac{al}{gt}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)dt)\,(1 + 2igq)}{(1 + (2i + 1)dl)\,1 + (2i + 2)aq)},$$

$$y_{12n-16} = r \left(\frac{bm}{hu}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)eu)\,(1 + 2ihr)}{(1 + (2i + 1)em)\,1 + (2i + 2)br)},$$

$$y_{12n-15} = s \left(\frac{cp}{kv}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 1)fv)\,(1 + 2iks)}{(1 + (2i + 1)fp)\,1 + (2i + 2)cs)},$$

$$y_{12n-14} = t^n \left(\frac{g}{al}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 2)dl)\,(1 + (2i + 1)aq)}{(1 + (2i + 1)gq)\,1 + (2i + 2)dt)},$$

$$y_{12n-13} = u^n \left(\frac{h}{bm}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 2)em)\,(1 + (2i + 1)br)}{(1 + (2i + 1)hr)\,1 + (2i + 2)eu)},$$

$$y_{12n-12} = v^n \left(\frac{k}{cp}\right)^{n-1} \prod_{i=0}^{n-2} \frac{(1 + (2i + 2)fp)\,(1 + (2i + 1)cs)}{(1 + (2i + 1)ks)\,1 + (2i + 2)fv)},$$

$$y_{12n-11} = \frac{d}{1+dl}\left(\frac{a}{t}\right)^{n-1}\left(\frac{l}{g}\right)^{n}\prod_{i=0}^{n-2}\frac{(1+(2i+1)dt)\,(1+(2i+2)gq)}{(1+(2i+2)aq)\,1+(2i+3)dl)},$$

$$y_{12n-10} = \frac{e}{1+em}\left(\frac{b}{u}\right)^{n-1}\left(\frac{m}{h}\right)^{n}\prod_{i=0}^{n-2}\frac{(1+(2i+1)eu)\,(1+(2i+2)hr)}{(1+(2i+2)br)\,1+(2i+1)em)},$$

$$y_{12n-9} = \frac{f}{1+fp}\left(\frac{c}{v}\right)^{n-1}\left(\frac{p}{k}\right)^{n}\prod_{i=0}^{n-2}\frac{(1+(2i+1)fv)\,(1+(2i+2)ks)}{(1+(2i+2)cs)\,1+(2i+1)fp)},$$

From system (1) that

$$\begin{aligned}
x_{12n-8} &= \frac{y_{12n-14}x_{12n-17}}{y_{12n-11}\left(y_{12n-14}x_{12n-17}\right)} \\
&= \frac{\begin{pmatrix} t^{n}\left(\frac{g}{al}\right)^{n-1}\prod_{i=0}^{n-2}\frac{(1+(2i+2)dl)(1+(2i+1)aq)}{(1+(2i+1)gq)(1+(2i+2)dt)} \\ \times d\left(\frac{al}{gt}\right)^{n-1}\prod_{i=0}^{n-2}\frac{(1+(2i+1)gq)(1+2idt)}{(1+(2i+2)dl)1+(2i+1)aq)} \end{pmatrix}}{\frac{d}{1+dl}\left(\frac{a}{t}\right)^{n-1}\left(\frac{l}{g}\right)^{n}\prod_{i=0}^{n-2}\frac{(1+(2i+1)dt)(1+(2i+2)gq)}{(1+(2i+2)aq)1+(2i+3)dl)}\begin{pmatrix} 1+t^{n}\left(\frac{g}{al}\right)^{n-1}\prod_{i=0}^{n-2}\frac{(1+(2i+2)dl)(1+(2i+1)aq)}{(1+(2i+1)gq)(1+(2i+2)dt)} \\ \times d\left(\frac{al}{gt}\right)^{n-1}\prod_{i=0}^{n-2}\frac{(1+(2i+1)gq)(1+2idt)}{(1+(2i+2)dl)1+(2i+1)aq)} \end{pmatrix}} \\
&= \frac{dt\prod_{i=0}^{n-2}\frac{(1+2idt)}{(1+(2i+2)dt)}}{\frac{d}{1+dl}\left(\frac{a}{t}\right)^{n-1}\left(\frac{l}{g}\right)^{n}\prod_{i=0}^{n-2}\frac{(1+(2i+1)dt)(1+(2i+2)gq)}{(1+(2i+2)aq)1+(2i+3)dl)}\left(1+dt\prod_{i=0}^{n-2}\frac{(1+2idt)}{(1+(2i+2)dt)}\right)} \\
&= \frac{t(1+dl)/(1+(2n-2)dt)}{\left(\frac{a}{t}\right)^{n-1}\left(\frac{l}{g}\right)^{n}\left(1+\frac{dt}{(1+(2n-2)dt)}\right)}\prod_{i=0}^{n-2}\frac{(1+(2i+2)aq)\,1+(2i+3)dl)}{(1+(2i+1)dt)\,(1+(2i+2)gq)} \\
&= \frac{(gt)^{n}\,(1+dl)}{a^{n-1}l^{n}\,(1+(2n-1)dt)}\prod_{i=0}^{n-2}\frac{(1+(2i+2)aq)\,1+(2i+3)dl)}{(1+(2i+1)dt)\,(1+(2i+2)gq)}.
\end{aligned}$$

Hence,

$$x_{12n-8} = \frac{(gt)^{n}}{a^{n-1}l^{n}}\prod_{i=0}^{n-1}\frac{(1+(2i+1)dl)\,(1+2iaq)}{(1+2igq)\,1+(2i+1)dt}.$$

Also, we can prove the other relations. This completes the proofs.

2.2 The Second System $x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}(1+y_{n-5}x_{n-8})},\ \ y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}(1-x_{n-5}y_{n-8})}$

In this part, we investigate the existence of the solutions for the following system of difference equations:

$$x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}\left(1+y_{n-5}x_{n-8}\right)},\ \ y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}\left(1-x_{n-5}y_{n-8}\right)}, \tag{2}$$

having nonzero real initial conditions $x_{-\alpha}$, $y_{-\alpha}$, $\alpha\in\{0,1,2,...8\}$, where $x_{-5}y_{-8}$, $x_{-4}y_{-7}$, $x_{-3}y_{-6}$, $x_{-2}y_{-5}$, $x_{-1}y_{-4}$, $x_0y_{-3}\neq 1$ and $x_{-8}y_{-5}$, $x_{-7}y_{-4}$, $x_{-6}y_{-3}$, $x_{-5}y_{-2}$, $x_{-4}y_{-1}$ and $x_{-3}y_0\neq -1$.

Theorem 2. Assume that $\{x_n,y_n\}$ is a solution for the system (2). Then for $n=0,1,...$ we obtain all solutions of system (2) are given by the following expressions

$$\begin{aligned}
x_{12n-8}&=(-1)^n\frac{(gt)^n(-1+dl)^n}{a^{n-1}l^n(1+dt)^n}, \quad x_{12n-7}=(-1)^n\frac{(hu)^n(-1+em)^n}{b^{n-1}m^n(1+eu)^n},\\
x_{12n-6}&=(-1)^n\frac{(kv)^n(-1+fp)^n}{c^{n-1}p^n(1+fv)^n}, \quad x_{12n-5}=(-1)^n d\frac{(al)^n(-1+gq)^n}{(gt)^n(1+aq)^n},\\
x_{12n-4}&=(-1)^n e\frac{(bm)^n(-1+hr)^n}{(hu)^n(1+br)^n}, \quad x_{12n-3}=(-1)^n f\frac{(cp)^n(-1+ks)^n}{(kv)^n(1+cs)^n},\\
x_{12n-2}&=(-1)^n\frac{t^ng^{n+1}(-1+dl)^n}{(al)^n(1+dt)^n}, \quad x_{12n-1}=(-1)^n\frac{u^nh^{n+1}(-1+em)^n}{(bm)^n(1+eu)^n},\\
x_{12n}&=(-1)^n\frac{v^nk^{n+1}(-1+fp)^n}{(cp)^n(1+fv)^n}, \quad x_{12n+1}=(-1)^n q\frac{l^na^{n+1}(-1+gq)^n}{g^nt^{n+1}(1+aq)^{n+1}},\\
x_{12n+2}&=(-1)^n r\frac{m^nb^{n+1}(-1+hr)^n}{h^nu^{n+1}(1+br)^{n+1}}, \quad x_{12n+3}=(-1)^n s\frac{p^nc^{n+1}(-1+ks)^n}{k^nv^{n+1}(1+cs)^{n+1}},\\
y_{12n-8}&=(-1)^n\frac{(gt)^n(1+aq)^n}{l^{n-1}a^n(-1+gq)^n}, \quad y_{12n-7}=(-1)^n\frac{(hu)^n(1+br)^n}{m^{n-1}b^n(-1+hr)^n},\\
y_{12n-6}&=(-1)^n\frac{(kv)^n(1+cs)^n}{p^{n-1}c^n(-1+ks)^n}, \quad y_{12n-5}=(-1)^n q\frac{(al)^n(1+dt)^n}{(gt)^n(-1+dl)^n},\\
y_{12n-4}&=(-1)^n r\frac{(bm)^n(1+eu)^n}{(hu)^n(-1+em)^n}, \quad y_{12n-3}=(-1)^n s\frac{(cp)^n(1+fv)^n}{(kv)^n(-1+fp)^n},\\
y_{12n-2}&=(-1)^n\frac{t^{n+1}g^n(1+aq)^n}{(al)^n(-1+gq)^n}, \quad y_{12n-1}=(-1)^n\frac{u^{n+1}h^n(1+br)^n}{(bm)^n(-1+hr)^n},\\
y_{12n}&=(-1)^n\frac{v^{n+1}k^n(1+cs)^n}{(cp)^n(-1+ks)^n}, \quad y_{12n+1}=(-1)^{n+1} d\frac{l^{n+1}a^n(1+dt)^n}{g^{n+1}t^n(-1+dl)^{n+1}},\\
y_{12n+2}&=(-1)^{n+1} e\frac{m^{n+1}b^n(1+eu)^n}{h^{n+1}u^n(-1+em)^{n+1}}, \quad x_{12n+3}=(-1)^{n+1} f\frac{p^{n+1}c^n(1+fv)^n}{k^{n+1}v^n(-1+fp)^{n+1}}.
\end{aligned}$$

Proof. For $n=0$ the result holds. Now suppose that $n>0$ and that our assumption holds for $n-1$.

That is,

$$x_{12n-20} = (-1)^{n-1} \frac{(gt)^{n-1}(-1+dl)^{n-1}}{a^{n-2}l^{n-1}(1+dt)^{n-1}}, \quad x_{12n-19} = (-1)^{n-1} \frac{(hu)^{n-1}(-1+em)^{n-1}}{b^{n-2}m^{n-1}(1+eu)^{n-1}},$$
$$x_{12n-18} = (-1)^{n-1} \frac{(kv)^{n-1}(-1+fp)^{n-1}}{c^{n-2}p^{n}-1(1+fv)^{n-1}}, \quad x_{12n-17} = (-1)^{n-1}\, d\frac{(al)^{n-1}(-1+gq)^{n-1}}{(gt)^{n-1}(1+aq)^{n-1}},$$
$$x_{12n-16} = (-1)^{n-1}\, e\frac{(bm)^{n-1}(-1+hr)^{n-1}}{(hu)^{n-1}(1+br)^{n-1}}, \quad x_{12n-15} = (-1)^{n-1}\, f\frac{(cp)^{n-1}(-1+ks)^{n-1}}{(kv)^{n-1}(1+cs)^{n-1}},$$
$$x_{12n-14} = (-1)^{n-1} \frac{t^{n-1}g^{n}(-1+dl)^{n-1}}{(al)^{n-1}(1+dt)^{n-1}}, \quad x_{12n-13} = (-1)^{n-1} \frac{u^{n-1}h^{n}(-1+em)^{n-1}}{(bm)^{n-1}(1+eu)^{n-1}},$$
$$x_{12n-12} = (-1)^{n-1} \frac{v^{n-1}k^{n}(-1+fp)^{n-1}}{(cp)^{n-1}(1+fv)^{n-1}}, \quad x_{12n-11} = (-1)^{n-1}\, q\frac{l^{n-1}a^{n}(-1+gq)^{n-1}}{g^{n-1}t^{n}(1+aq)^{n}},$$
$$x_{12n-10} = (-1)^{n-1}\, r\frac{m^{n-1}b^{n}(-1+hr)^{n-1}}{h^{n-1}u^{n}(1+br)^{n}}, \quad x_{12n-9} = (-1)^{n-1}\, s\frac{p^{n-1}c^{n}(-1+ks)^{n-1}}{k^{n-1}v^{n}(1+cs)^{n}},$$
$$y_{12n-20} = (-1)^{n-1} \frac{(gt)^{n-1}(1+aq)^{n-1}}{l^{n-2}a^{n-1}(-1+gq)^{n-1}}, \quad y_{12n-19} = (-1)^{n-1} \frac{(hu)^{n-1}(1+br)^{n-1}}{m^{n-2}b^{n-1}(-1+hr)^{n-1}},$$
$$y_{12n-18} = (-1)^{n-1} \frac{(kv)^{n-1}(1+cs)^{n-1}}{p^{n-2}c^{n-1}(-1+ks)^{n-1}}, \quad y_{12n-17} = (-1)^{n-1}\, q\frac{(al)^{n-1}(1+dt)^{n-1}}{(gt)^{n-1}(-1+dl)^{n-1}},$$
$$y_{12n-16} = (-1)^{n-1}\, r\frac{(bm)^{n-1}(1+eu)^{n-1}}{(hu)^{n-1}(-1+em)^{n-1}}, \quad y_{12n-15} = (-1)^{n-1}\, s\frac{(cp)^{n-1}(1+fv)^{n-1}}{(kv)^{n-1}(-1+fp)^{n-1}},$$
$$y_{12n-14} = (-1)^{n-1} \frac{t^{n}g^{n-1}(1+aq)^{n-1}}{(al)^{n-1}(-1+gq)^{n-1}}, \quad y_{12n-13} = (-1)^{n-1} \frac{u^{n}h^{n-1}(1+br)^{n-1}}{(bm)^{n-1}(-1+hr)^{n-1}},$$
$$y_{12n-12} = (-1)^{n-1} \frac{v^{n}k^{n-1}(1+cs)^{n-1}}{(cp)^{n-1}(-1+ks)^{n-1}}, \quad y_{12n-11} = (-1)^{n}\, d\frac{l^{n}a^{n-1}(1+dt)^{n-1}}{g^{n}t^{n-1}(-1+dl)^{n}},$$
$$y_{12n-10} = (-1)^{n}\, e\frac{m^{n}b^{n-1}(1+eu)^{n-1}}{h^{n}u^{n-1}(-1+em)^{n}}, \quad x_{12n-9} = (-1)^{n}\, f\frac{p^{n}c^{n-1}(1+fv)^{n-1}}{k^{n}v^{n-1}(-1+fp)^{n}},$$

Deducing system (2) we get

$$\begin{aligned} y_{12n-8} &= \frac{x_{12n-14}y_{12n-17}}{x_{12n-11}\left(1-x_{12n-14}y_{12n-17}\right)} \\ &= \frac{\left((-1)^{n-1}\frac{t^{n-1}g^{n}(-1+dl)^{n-1}}{(al)^{n-1}(1+dt)^{n-1}}\right)\left((-1)^{n-1}\, q\frac{(al)^{n-1}(1+dt)^{n-1}}{(gt)^{n-1}(-1+dl)^{n-1}}\right)}{\begin{array}{c}(-1)^{n-1}\, q\frac{l^{n-1}a^{n}(-1+gq)^{n-1}}{g^{n-1}t^{n}(1+aq)^{n}} \\ \left(1-\left((-1)^{n-1}\frac{t^{n-1}g^{n}(-1+dl)^{n-1}}{(al)^{n-1}(1+dt)^{n-1}}\right)\times\left((-1)^{n-1}\, q\frac{(al)^{n-1}(1+dt)^{n-1}}{(gt)^{n-1}(-1+dl)^{n-1}}\right)\right)\end{array}} \\ &= \frac{(-1)^{2n}(gt)^{n}(1+aq)^{n}}{(-1)^{n-1}l^{n-1}a^{n}(-1+gq)^{n-1}(1-gq)} = (-1)^{n}\frac{(gt)^{n}(1+aq)^{n}}{l^{n-1}a^{n}(-1+gq)^{n}} \end{aligned}$$

Also, we can prove the other relations. Thus the proof is completed.

2.3 The Third System $x_{n+1} = \frac{y_{n-5}x_{n-8}}{y_{n-2}(1+y_{n-5}x_{n-8})}$, $y_{n+1} = \frac{x_{n-5}y_{n-8}}{x_{n-2}(-1+x_{n-5}y_{n-8})}$.

In this section, we study the dynamics of the solutions for the following system of difference equations

$$x_{n+1} = \frac{y_{n-5}x_{n-8}}{y_{n-2}\left(1 + y_{n-5}x_{n-8}\right)}, \quad y_{n+1} = \frac{x_{n-5}y_{n-8}}{x_{n-2}\left(-1 + x_{n-5}y_{n-8}\right)}, \tag{3}$$

with nonzero real initial conditions $x_{-\alpha}$, $y_{-\alpha}$, $\alpha \in \{0, 1, 2, ...8\}$, where $x_{-5}y_{-8}$, $x_{-4}y_{-7}$, $x_{-3}y_{-6}$, $x_{-2}y_{-5}$, $x_{-1}y_{-4}$, $x_0y_{-3} \neq 1, \neq \frac{1}{2}$ and $x_{-8}y_{-5}$, $x_{-7}y_{-4}$, $x_{-6}y_{-3}$, $x_{-5}y_{-2}$, $x_{-4}y_{-1}$, $x_{-3}y_0 \neq \pm 1$.

Theorem 3. If $\{x_n, y_n\}$ are solutions of difference equation systems (3). Then for $n = 0, 1, ...$

$$x_{24n-8} = \frac{(gt)^{2n}(-1+dl)^{2n}}{a^{2n-1}l^{2n}(-1+2gq)^n(-1+dt)^n(1+dt)^n},$$
$$x_{24n-7} = \frac{(hu)^{2n}(-1+em)^{2n}}{b^{2n-1}m^{2n}(-1+2hr)^n(-1+eu)^n(1+eu)^n},$$
$$x_{24n-6} = \frac{(kv)^{2n}(-1+fp)^{2n}}{c^{2n-1}p^{2n}(-1+2ks)^n(-1+fv)^n(1+fv)^n},$$
$$x_{24n-5} = d\frac{(al)^{2n}(-1+gq)^{2n}}{(gt)^{2n}(-1+2dl)^n(-1+aq)^n(1+aq)^n},$$
$$x_{24n-4} = e\frac{(bm)^{2n}(-1+hr)^{2n}}{(hu)^{2n}(-1+2em)^n(-1+br)^n(1+br)^n},$$
$$x_{24n-3} = f\frac{(cp)^{2n}(-1+ks)^{2n}}{(kv)^{2n}(-1+2fp)^n(-1+cs)^n(1+cs)^n},$$
$$x_{24n-2} = \frac{t^{2n}g^{2n+1}(-1+dl)^{2n}}{(al)^{2n}(-1+2gq)^n(-1+dt)^n(1+dt)^n},$$
$$x_{24n-1} = \frac{u^{2n}h^{2n+1}(-1+em)^{2n}}{(bm)^{2n}(-1+2hr)^n(-1+eu)^n(1+eu)^n},$$
$$x_{24n} = \frac{v^{2n}k^{2n+1}(-1+fp)^{2n}}{(cp)^{2n}(-1+2ks)^n(-1+fv)^n(1+fv)^n},$$
$$x_{24n+1} = q\frac{l^{2n}a^{2n+1}(-1+gq)^{2n}}{g^{2n}t^{2n+1}(-1+2dl)^n(-1+aq)^n(1+aq)^{n+1}},$$
$$x_{24n+2} = r\frac{m^{2n}b^{2n+1}(-1+hr)^{2n}}{h^{2n}u^{2n+1}(-1+2em)^n(-1+br)^n(1+br)^{n+1}},$$
$$x_{24n+3} = s\frac{p^{2n}c^{2n+1}(-1+ks)^{2n}}{k^{2n}v^{2n+1}(-1+2fp)^n(-1+cs)^n(1+cs)^{n+1}},$$
$$x_{24n+4} = \frac{(gt)^{2n+1}(-1+dl)^{2n+1}}{a^{2n}l^{2n+1}(-1+2gq)^n(-1+dt)^n(1+dt)^{n+1}},$$
$$x_{24n+5} = \frac{(hu)^{2n+1}(-1+em)^{2n+1}}{b^{2n}m^{2n+1}(-1+2hr)^n(-1+eu)^n(1+eu)^{n+1}},$$

$$x_{24n+6} = \frac{(kv)^{2n+1}(-1+fp)^{2n+1}}{c^{2n}p^{2n+1}(-1+2ks)^{n}(-1+fv)^{n}(1+fv)^{n+1}},$$
$$x_{24n+7} = (-1)^{n}d\frac{(al)^{2n+1}(-1+gq)^{2n+1}}{(gt)^{2n+1}(-1+2dl)^{n+1}(-1+aq)^{n}(1+aq)^{n+1}},$$
$$x_{24n+8} = (-1)^{n}e\frac{(bm)^{2n+1}(-1+hr)^{2n+1}}{(hu)^{2n+1}(-1+2em)^{n+1}(-1+br)^{n}(1+br)^{n+1}},$$
$$x_{24n+9} = (-1)^{n}f\frac{(cp)^{2n+1}(-1+ks)^{2n+1}}{(kv)^{2n+1}(-1+2fp)^{n+1}(-1+cs)^{n}(1+cs)^{n+1}},$$
$$x_{24n+10} = (-1)^{n+1}\frac{t^{2n+1}g^{2n+2}(-1+dl)^{2n+1}}{(al)^{2n+1}(-1+2gq)^{n+1}(-1+dt)^{n}(1+dt)^{n+1}},$$
$$x_{24n+11} = (-1)^{n+1}\frac{u^{2n+1}h^{2n+2}(-1+em)^{2n+1}}{(bm)^{2n+1}(-1+2hr)^{n+1}(-1+eu)^{n}(1+eu)^{n+1}},$$
$$x_{24n+12} = (-1)^{n+1}\frac{v^{2n+1}k^{2n+2}(-1+fp)^{2n+1}}{(cp)^{2n+1}(-1+2ks)^{n+1}(-1+fv)^{n}(1+fv)^{n+1}},$$
$$x_{24n+13} = (-1)^{n+1}q\frac{l^{2n+1}a^{2n+2}(-1+gq)^{2n+1}}{g^{2n+1}t^{2n+2}(-1+2dl)^{n+1}(-1+aq)^{n+1}(1+aq)^{n+1}},$$
$$x_{24n+14} = (-1)^{n+1}r\frac{m^{2n+1}b^{2n+2}(-1+hr)^{2n+1}}{h^{2n+1}u^{2n+2}(-1+2em)^{n+1}(-1+br)^{n+1}(1+br)^{n+1}},$$
$$x_{24n+15} = (-1)^{n+1}s\frac{p^{2n+1}c^{2n+2}(-1+ks)^{2n+1}}{k^{2n+1}v^{2n+2}(-1+2fp)^{n+1}(-1+cs)^{n+1}(1+cs)^{n+1}},$$
$$y_{24n-8} = \frac{(gt)^{2n}(-1+2dl)^{n}(-1+aq)^{n}(1+aq)^{n}}{l^{2n-1}a^{2n}(-1+gq)^{2n}},$$
$$y_{24n-7} = \frac{(hu)^{2n}(-1+2em)^{n}(-1+br)^{n}(1+br)^{n}}{m^{2n-1}b^{2n}(-1+hr)^{2n}},$$
$$y_{24n-6} = \frac{(kv)^{2n}(-1+2fp)^{n}(-1+cs)^{n}(1+cs)^{n}}{p^{2n-1}c^{2n}(-1+ks)^{2n}},$$
$$y_{24n-5} = q\frac{(al)^{2n}(-1+2gq)^{n}(-1+dt)^{n}(1+dt)^{n}}{(gt)^{2n}(-1+dl)^{2n}},$$
$$y_{24n-4} = r\frac{(bm)^{2n}(-1+2hr)^{n}(-1+eu)^{n}(1+eu)^{n}}{(hu)^{2n}(-1+em)^{2n}},$$
$$y_{24n-3} = s\frac{(cp)^{2n}(-1+2ks)^{n}(-1+fv)^{n}(1+fv)^{n}}{(kv)^{2n}(-1+fp)^{2n}},$$
$$y_{24n-2} = \frac{g^{2n}t^{2n+1}(-1+2dl)^{n}(-1+aq)^{n}(1+aq)^{n}}{(al)^{2n}(-1+gq)^{2n}},$$
$$y_{24n-1} = \frac{h^{2n}u^{2n+1}(-1+2em)^{n}(-1+br)^{n}(1+br)^{n}}{(bm)^{2n}(-1+hr)^{2n}},$$

$$
\begin{aligned}
y_{24n} &= \frac{k^{2n}v^{2n+1}(-1+2fp)^n(-1+cs)^n(1+cs)^n}{(cp)^{2n}(-1+ks)^{2n}},\\
y_{24n+1} &= d\frac{a^{2n}l^{2n+1}(-1+2gq)^n(-1+dt)^n(1+dt)^n}{t^{2n}g^{2n+1}(-1+dl)^{2n+1}},\\
y_{24n+2} &= e\frac{b^{2n}m^{2n+1}(-1+2hr)^n(-1+eu)^n(1+eu)^n}{u^{2n}h^{2n+1}(-1+em)^{2n+1}},\\
y_{24n+3} &= f\frac{c^{2n}p^{2n+1}(-1+2ks)^n(-1+fv)^n(1+fv)^n}{v^{2n}k^{2n+1}(-1+fp)^{2n+1}},\\
y_{24n+4} &= \frac{(gt)^{2n+1}(-1+2dl)^n(-1+aq)^n(1+aq)^{n+1}}{l^{2n}a^{2n+1}(-1+gq)^{2n+1}},\\
y_{24n+5} &= \frac{(hu)^{2n+1}(-1+2em)^n(-1+br)^n(1+br)^{n+1}}{m^{2n}b^{2n+1}(-1+hr)^{2n+1}},\\
y_{24n+6} &= \frac{(kv)^{2n+1}(-1+2fp)^n(-1+cs)^n(1+cs)^{n+1}}{p^{2n}c^{2n+1}(-1+ks)^{2n+1}},\\
y_{24n+7} &= (-1)^n q\frac{(al)^{2n+1}(-1+2gq)^n(-1+dt)^n(1+dt)^{n+1}}{(gt)^{2n+1}(-1+dl)^{2n+1}},\\
y_{24n+8} &= (-1)^n r\frac{(bm)^{2n+1}(-1+2hr)^n(-1+eu)^n(1+eu)^{n+1}}{(hu)^{2n+1}(-1+em)^{2n+1}},\\
y_{24n+9} &= (-1)^n s\frac{(cp)^{2n+1}(-1+2ks)^n(-1+fv)^n(1+fv)^{n+1}}{(kv)^{2n+1}(-1+fp)^{2n+1}},\\
y_{24n+10} &= (-1)^{n+1}\frac{g^{2n+1}t^{2n+2}(-1+2dl)^{n+1}(-1+aq)^n(1+aq)^{n+1}}{(al)^{2n+1}(-1+gq)^{2n+1}},\\
y_{24n+11} &= (-1)^{n+1}\frac{h^{2n+1}u^{2n+2}(-1+2em)^{n+1}(-1+br)^n(1+br)^{n+1}}{(bm)^{2n+1}(-1+hr)^{2n+1}},\\
y_{24n+12} &= (-1)^{n+1}\frac{k^{2n+1}v^{2n+2}(-1+2fp)^{n+1}(-1+cs)^n(1+cs)^{n+1}}{(cp)^{2n+1}(-1+ks)^{2n+1}},\\
y_{24n+13} &= d\frac{a^{2n+1}l^{2n+2}(-1+2gq)^{n+1}(-1+dt)^n(1+dt)^{n+1}}{t^{2n+1}g^{2n+2}(-1+dl)^{2n+2}},\\
y_{24n+14} &= e\frac{b^{2n+1}m^{2n+2}(-1+2hr)^{n+1}(-1+eu)^n(1+eu)^{n+1}}{u^{2n+1}h^{2n+2}(-1+em)^{2n+2}},\\
y_{24n+15} &= f\frac{c^{2n+1}p^{2n+2}(-1+2ks)^{n+1}(-1+fv)^n(1+fv)^{n+1}}{v^{2n+1}k^{2n+2}(-1+fp)^{2n+2}}.
\end{aligned}
$$

Proof. **For** $n = 0$ the result holds. Now assume that $n > 0$ and that our assumption holds for $n - 1$, that is,

$$x_{24n-32} = \frac{(gt)^{2n-2}(-1+dl)^{2n-2}}{a^{2n-3}l^{2n-2}(-1+2gq)^{n-1}(-1+dt)^{n-1}(1+dt)^{n-1}},$$
$$x_{24n-31} = \frac{(hu)^{2n-2}(-1+em)^{2n-2}}{b^{2n-3}m^{2n-2}(-1+2hr)^{n-1}(-1+eu)^{n-1}(1+eu)^{n-1}},$$
$$x_{24n-30} = \frac{(kv)^{2n-2}(-1+fp)^{2n-2}}{c^{2n-3}p^{2n-1}(-1+2ks)^{n-1}(-1+fv)^{n-1}(1+fv)^{n-1}},$$
$$x_{24n-29} = d\frac{(al)^{2n-2}(-1+gq)^{2n-2}}{(gt)^{2n-2}(-1+2dl)^{n-1}(-1+aq)^{n-1}(1+aq)^{n-1}},$$
$$x_{24n-28} = e\frac{(bm)^{2n-1}(-1+hr)^{2n-1}}{(hu)^{2n-2}(-1+2em)^{n-1}(-1+br)^{n-1}(1+br)^{n-1}},$$
$$x_{24n-27} = f\frac{(cp)^{2n-2}(-1+ks)^{2n-2}}{(kv)^{2n-2}(-1+2fp)^{n-1}(-1+cs)^{n-1}(1+cs)^{n-1}},$$
$$x_{24n-26} = \frac{t^{2n-2}g^{2n-1}(-1+dl)^{2n-2}}{(al)^{2n-2}(-1+2gq)^{n-1}(-1+dt)^{n-1}(1+dt)^{n-1}},$$
$$x_{24n-25} = \frac{u^{2n-2}h^{2n-1}(-1+em)^{2n-2}}{(bm)^{2n-2}(-1+2hr)^{n-1}(-1+eu)^{n-1}(1+eu)^{n-1}},$$
$$x_{24n-24} = \frac{v^{2n-2}k^{2n-1}(-1+fp)^{2n-2}}{(cp)^{2n-2}(-1+2ks)^{n-1}(-1+fv)^{n-1}(1+fv)^{n-1}},$$
$$x_{24n-23} = q\frac{l^{2n-2}a^{2n-1}(-1+gq)^{2n-2}}{g^{2n-2}t^{2n-1}(-1+2dl)^{n-1}(-1+aq)^{n-1}(1+aq)^{n}},$$
$$x_{24n-22} = r\frac{m^{2n-2}b^{2n-1}(-1+hr)^{2n-2}}{h^{2n-2}u^{2n-1}(-1+2em)^{n-1}(-1+br)^{n-1}(1+br)^{n}},$$
$$x_{24n-21} = s\frac{p^{2n-2}c^{2n-1}(-1+ks)^{2n-2}}{k^{2n-2}v^{2n-1}(-1+2fp)^{n-1}(-1+cs)^{n-1}(1+cs)^{n}},$$
$$x_{24m-20} = \frac{(gt)^{2n-1}(-1+dl)^{2n-1}}{a^{2n-2}l^{2n-1}(-1+2gq)^{n-1}(-1+dt)^{n-1}(1+dt)^{n}},$$
$$x_{24n-19} = \frac{(hu)^{2n-1}(-1+em)^{2n-1}}{b^{2n-2}m^{2n-1}(-1+2hr)^{n-1}(-1+eu)^{n-1}(1+eu)^{n}},$$
$$x_{24n-18} = \frac{(kv)^{2n-1}(-1+fp)^{2n-1}}{c^{2n-2}p^{2n-1}(-1+2ks)^{n-1}(-1+fv)^{n-1}(1+fv)^{n}},$$
$$x_{24n-17} = (-1)^{n-1}d\frac{(al)^{2n-1}(-1+gq)^{2n-1}}{(gt)^{2n-1}(-1+2dl)^{n}(-1+aq)^{n-1}(1+aq)^{n}},$$
$$x_{24n-16} = (-1)^{n-1}e\frac{(bm)^{2n-1}(-1+hr)^{2n-1}}{(hu)^{2n-1}(-1+2em)^{n}(-1+br)^{n-1}(1+br)^{n}},$$
$$x_{24n-15} = (-1)^{n-1}f\frac{(cp)^{2n-1}(-1+ks)^{2n-1}}{(kv)^{2n-1}(-1+2fp)^{n}(-1+cs)^{n-1}(1+cs)^{n}},$$

$$x_{24n-14}=(-1)^n\frac{t^{2n-1}g^{2n}(-1+dl)^{2n-1}}{(al)^{2n-1}(-1+2gq)^n(-1+dt)^{n-1}(1+dt)^n},$$
$$x_{24n-13}=(-1)^n\frac{u^{2n-1}h^{2n}(-1+em)^{2n-1}}{(bm)^{2n-1}(-1+2hr)^n(-1+eu)^{n-1}(1+eu)^n},$$
$$x_{24n-12}=(-1)^n\frac{v^{2n-1}k^{2n}(-1+fp)^{2n-1}}{(cp)^{2n-1}(-1+2ks)^n(-1+fv)^{n-1}(1+fv)^n},$$
$$x_{24n-11}=(-1)^n q\frac{l^{2n-1}a^{2n}(-1+gq)^{2n-1}}{g^{2n-1}t^{2n}(-1+2dl)^n(-1+aq)^n(1+aq)^n},$$
$$x_{24n-10}=(-1)^n r\frac{m^{2n-1}b^{2n}(-1+hr)^{2n-1}}{h^{2n-1}u^{2n}(-1+2em)^n(-1+br)^n(1+br)^n},$$
$$x_{24n-9}=(-1)^n s\frac{p^{2n-1}c^{2n}(-1+ks)^{2n-1}}{k^{2n-1}v^{2n}(-1+2fp)^n(-1+cs)^n(1+cs)^n},$$
$$y_{24n-32}=\frac{(gt)^{2n-2}(-1+2dl)^{n-1}(-1+aq)^{n-1}(1+aq)^{n-1}}{l^{2n-3}a^{2n-2}(-1+gq)^{2n-2}},$$
$$y_{24n-31}=\frac{(hu)^{2n-2}(-1+2em)^{n-1}(-1+br)^{n-1}(1+br)^{n-1}}{m^{2n-3}b^{2n-2}(-1+gq)^{2n-2}},$$
$$y_{24n-30}=\frac{(kv)^{2n-2}(-1+2fp)^{n-1}(-1+cs)^{n-1}(1+cs)^{n-1}}{p^{2n-3}c^{2n-2}(-1+ks)^{2n-2}},$$
$$y_{24n-29}=q\frac{(al)^{2n-2}(-1+2gq)^{n-1}(-1+dt)^{n-1}(1+dt)^{n-1}}{(gt)^{2n-2}(-1+dl)^{2n-2}},$$
$$y_{24n-28}=r\frac{(bm)^{2n-2}(-1+2hr)^{n-1}(-1+eu)^{n-1}(1+eu)^{n-1}}{(hu)^{2n-2}(-1+em)^{2n-2}},$$
$$y_{24n-27}=s\frac{(cp)^{2n-2}(-1+2ks)^{n-1}(-1+fv)^{n-1}(1+fv)^{n-1}}{(kv)^{2n-2}(-1+fp)^{2n-2}},$$
$$y_{24n-26}=\frac{g^{2n-2}t^{2n-1}(-1+2dl)^{n-1}(-1+aq)^{n-1}(1+aq)^{n-1}}{(al)^{2n-2}(-1+gq)^{2n-2}},$$
$$y_{24n-25}=\frac{h^{2n-2}u^{2n-1}(-1+2em)^{n-1}(-1+br)^{n-1}(1+br)^{n-1}}{(bm)^{2n-2}(-1+hr)^{2n-2}},$$
$$y_{24n-24}=\frac{k^{2n-2}v^{2n-1}(-1+2fp)^{n-1}(-1+cs)^{n-1}(1+cs)^{n-1}}{(cp)^{2n-2}(-1+ks)^{2n-2}},$$
$$y_{24n-23}=d\frac{a^{2n-2}l^{2n-1}(-1+2gq)^{n-1}(-1+dt)^{n-1}(1+dt)^{n-1}}{t^{2n-2}g^{2n-1}(-1+dl)^{2n-1}},$$
$$y_{24n-22}=e\frac{b^{2n-2}m^{2n-1}(-1+2hr)^{n-1}(-1+eu)^{n-1}(1+eu)^{n-1}}{u^{2n-2}h^{2n-1}(-1+em)^{2n-1}},$$
$$y_{24n-21}=f\frac{c^{2n-2}p^{2n-1}(-1+2ks)^{n-1}(-1+fv)^{n-1}(1+fv)^{n-1}}{v^{2n-2}k^{2n-1}(-1+fp)^{2n-1}},$$
$$y_{24n-20}=\frac{(gt)^{2n-1}(-1+2dl)^{n-1}(-1+aq)^{n-1}(1+aq)^{n}}{l^{2n-2}a^{2n-1}(-1+gq)^{2n-1}},$$

$$y_{24n-19} = \frac{(hu)^{2n-1}(-1+2em)^{n-1}(-1+br)^{n-1}(1+br)^n}{m^{2n-2}b^{2n-1}(-1+hr)^{2n-1}},$$
$$y_{24n-18} = \frac{(kv)^{2n-1}(-1+2fp)^{n-1}(-1+cs)^{n-1}(1+cs)^n}{p^{2n-2}c^{2n-1}(-1+ks)^{2n-1}},$$
$$y_{24n-17} = (-1)^{n-1}q\frac{(al)^{2n-1}(-1+2gq)^{n-1}(-1+dt)^{n-1}(1+dt)^n}{(gt)^{2n-1}(-1+dl)^{2n-1}},$$
$$y_{24n-16} = (-1)^{n-1}r\frac{(bm)^{2n-1}(-1+2hr)^{n-1}(-1+eu)^{n-1}(1+eu)^n}{(hu)^{2n-1}(-1+em)^{2n-1}},$$
$$y_{24n-15} = (-1)^{n-1}s\frac{(cp)^{2n-1}(-1+2ks)^{n-1}(-1+fv)^{n-1}(1+fv)^n}{(kv)^{2n-1}(-1+fp)^{2n-1}},$$
$$y_{24n-14} = (-1)^n\frac{g^{2n-1}t^{2n}(-1+2dl)^n(-1+aq)^{n-1}(1+aq)^n}{(al)^{2n-1}(-1+gq)^{2n-1}},$$
$$y_{24n-13} = (-1)^n\frac{h^{2n-1}u^{2n}(-1+2em)^n(-1+br)^{n-1}(1+br)^n}{(bm)^{2n-1}(-1+hr)^{2n-1}},$$
$$y_{24n-12} = (-1)^n\frac{k^{2n-1}v^{2n}(-1+2fp)^n(-1+cs)^{n-1}(1+cs)^n}{(cp)^{2n-1}(-1+ks)^{2n-1}},$$
$$y_{24n-11} = d\frac{a^{2n-1}l^{2n}(-1+2gq)^n(-1+dt)^{n-1}(1+dt)^n}{t^{2n-1}g^{2n}(-1+dl)^{2n}},$$
$$y_{24n-10} = e\frac{b^{2n-1}m^{2n}(-1+2hr)^n(-1+eu)^{n-1}(1+eu)^n}{u^{2n-1}h^{2n}(-1+em)^{2n}},$$
$$y_{24n-9} = f\frac{c^{2n-1}p^{2n}(-1+2ks)^n(-1+fv)^{n-1}(1+fv)^n}{v^{2n-1}k^{2n}(-1+fp)^{2n}}.$$

Deducing from system (3) we get,

$$\begin{aligned}
x_{24n-8} &= \frac{y_{24n-14}x_{24n-17}}{y_{24n-11}(1+y_{24n-14}x_{24n-17})} \\
&= \frac{\left(\begin{array}{c}(-1)^n\frac{g^{2n-1}t^{2n}(-1+2dl)^n(-1+aq)^{n-1}(1+aq)^n}{(al)^{2n-1}(-1+gq)^{2n-1}} \\ \times(-1)^{n-1}d\frac{(al)^{2n-1}(-1+gq)^{2n-1}}{(gt)^{2n-1}(-1+2dl)^n(-1+aq)^{n-1}(1+aq)^n}\end{array}\right)}{d\frac{a^{2n-1}l^{2n}(-1+2gq)^n(-1+dt)^{n-1}(1+dt)^n}{t^{2n-1}g^{2n}(-1+dl)^{2n}}\left(1+\left(\begin{array}{c}(-1)^n\frac{g^{2n-1}t^{2n}(-1+2dl)^n(-1+aq)^{n-1}(1+aq)^n}{(al)^{2n-1}(-1+gq)^{2n-1}} \\ \times(-1)^{n-1}d\frac{(al)^{2n-1}(-1+gq)^{2n-1}}{(gt)^{2n-1}(-1+2dl)^n(-1+aq)^{n-1}(1+aq)^n}\end{array}\right)\right)} \\
&= \frac{-t^{2n}g^{2n}(-1+dl)^{2n}}{a^{2n-1}l^{2n}(-1+2gq)^n(-1+dt)^{n-1}(1+dt)^n\,(1-dt)} \\
&= \frac{t^{2n}g^{2n}(-1+dl)^{2n}}{a^{2n-1}l^{2n}(-1+2gq)^n(-1+dt)^n(1+dt)^n}.
\end{aligned}$$

Similarly, we can prove the other relations.

2.4 The Fourth System $x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}(1+y_{n-5}x_{n-8})},\ y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}(-1-x_{n-5}y_{n-8})}$.

In this subsection, we get the solution of the following system of the difference equations

$$x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}\left(1+y_{n-5}x_{n-8}\right)},\quad y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}\left(-1-x_{n-5}y_{n-8}\right)},\tag{4}$$

with nonzero real initial conditions where $x_{-5}y_{-8}, x_{-4}y_{-7}, x_{-3}y_{-6}, x_{-2}y_{-5}, x_{-1}y_{-4}, x_{0}y_{-3}\neq\pm1$ and $x_{-8}y_{-5}, x_{-7}y_{-4}, x_{-6}y_{-3},\ x_{-5}y_{-2},\ x_{-4}y_{-1}, x_{-3}y_{0}\neq-1,\neq-\frac{1}{2}$.

Theorem 4. Assume that $\{x_n, y_n\}$ are solutions of system

$$x_{n+1}=\frac{y_{n-5}x_{n-8}}{y_{n-2}\left(1+y_{n-5}x_{n-8}\right)},\quad y_{n+1}=\frac{x_{n-5}y_{n-8}}{x_{n-2}\left(-1-x_{n-5}y_{n-8}\right)},$$

then for $n=0,1,\ldots$, we see that

$$\begin{aligned}
x_{24n-8}&=(-1)^n\frac{(gt)^{2n}(-1+dl)^n(1+dl)^n(1+2aq)^n}{a^{2n-1}l^{2n}(1+dt)^{2n}},\\
x_{24n-7}&=(-1)^n\frac{(hu)^{2n}(-1+em)^n(1+em)^n(1+2br)^n}{b^{2n-1}m^{2n}(1+eu)^{2n}},\\
x_{24n-6}&=(-1)^n\frac{(kv)^{2n}(-1+fp)^n(1+fp)^n(1+2cs)^n}{c^{2n-1}p^{2n}(1+fv)^{2n}},\\
x_{24n-5}&=(-1)^n d\frac{(al)^{2n}(-1+gq)^n(1+gq)^n(1+2dt)^n}{(gt)^{2n}(1+aq)^{2n}},\\
x_{24n-4}&=(-1)^n e\frac{(bm)^{2n}(-1+hr)^n(1+hr)^n(1+2eu)^n}{(hu)^{2n}(1+br)^{2n}},\\
x_{24n-3}&=(-1)^n f\frac{(cp)^{2n}(-1+ks)^n(1+ks)^n(1+2fv)^n}{(kv)^{2n}(1+cs)^{2n}},\\
x_{24n-2}&=(-1)^n\frac{t^{2n}g^{2n+1}(-1+dl)^n(1+dl)^n(1+2aq)^n}{(al)^{2n}(1+dt)^{2n}},\\
x_{24n-1}&=(-1)^n\frac{u^{2n}h^{2n+1}(-1+em)^n(1+em)^n(1+2br)^n}{(bm)^{2n}(1+eu)^{2n}},\\
x_{24n}&=(-1)^n\frac{v^{2n}k^{2n+1}(-1+fp)^n(1+fp)^n(1+2cs)^n}{(cp)^{2n}(1+fv)^{2n}},\\
x_{24n+1}&=(-1)^n q\frac{l^{2n}a^{2n+1}(-1+gq)^n(1+gq)^n(1+2dt)^n}{g^{2n}t^{2n+1}(1+aq)^{2n+1}},
\end{aligned}$$

$$x_{24n+2} = (-1)^n r\frac{m^{2n}b^{2n+1}(-1+hr)^n(-1+hr)^n(1+2eu)^n}{h^{2n}u^{2n+1}(1+br)^{2n+1}},$$
$$x_{24n+3} = (-1)^n s\frac{p^{2n}c^{2n+1}(-1+ks)^n(1+ks)^n(1+2fv)^n}{k^{2n}v^{2n+1}(1+cs)^{2n+1}},$$
$$x_{24n+4} = (-1)^{n+1}\frac{(gt)^{2n+1}(-1+dl)^n(1+dl)^{n+1}(1+2aq)^n}{a^{2n}l^{2n+1}(1+dt)^{2n+1}},$$
$$x_{24n+5} = (-1)^{n+1}\frac{(hu)^{2n+1}(-1+em)^n(1+em)^{n+1}(1+2br)^n}{b^{2n}m^{2n+1}(1+eu)^{2n+1}},$$
$$x_{24n+6} = (-1)^{n+1}\frac{(kv)^{2n+1}(-1+fp)^n(1+fp)^{n+1}(1+2cs)^n}{c^{2n}p^{2n+1}(1+fv)^{2n+1}},$$
$$x_{24n+7} = (-1)^n d\frac{(al)^{2n+1}(-1+gq)^n(1+gq)^{n+1}(1+2dt)^n}{(gt)^{2n+1}(1+aq)^{2n+1}},$$
$$x_{24n+8} = (-1)^n e\frac{(bm)^{2n+1}(-1+hr)^n(1+hr)^{n+1}(1+2eu)^n}{(hu)^{2n+1}(1+br)^{2n+1}},$$
$$x_{24n+9} = (-1)^n f\frac{(cp)^{2n+1}(-1+ks)^n(1+ks)^{n+1}(1+2fv)^n}{(kv)^{2n+1}(1+cs)^{2n+1}},$$
$$x_{24n+10} = (-1)^{n+1}\frac{t^{2n+1}g^{2n+2}(-1+dl)^n(1+dl)^{n+1}(1+2aq)^{n+1}}{(al)^{2n+1}(1+dt)^{2n+1}},$$
$$x_{24n+11} = (-1)^{n+1}\frac{u^{2n+1}h^{2n+2}(-1+em)^n(1+em)^{n+1}(1+2br)^{n+1}}{(bm)^{2n+1}(1+eu)^{2n+1}},$$
$$x_{24n+12} = (-1)^{n+1}\frac{v^{2n+1}k^{2n+2}(-1+fp)^n(1+fp)^{n+1}(1+2cs)^{n+1}}{(cp)^{2n+1}(1+fv)^{2n+1}},$$
$$x_{24n+13} = (-1)^n q\frac{l^{2n+1}a^{2n+2}(-1+gq)^n(1+gq)^{n+1}(1+2dt)^{n+1}}{g^{2n+1}t^{2n+2}(1+aq)^{2n+2}},$$
$$x_{24n+14} = (-1)^n r\frac{m^{2n+1}b^{2n+2}(-1+hr)^n(1+hr)^{n+1}(1+2eu)^{n+1}}{h^{2n+1}u^{2n+2}(1+br)^{2n+2}},$$
$$x_{24n+15} = (-1)^{n+1} s\frac{p^{2n+1}c^{2n+2}(-1+ks)^n(1+ks)^{n+1}(1+2fv)^{n+1}}{k^{2n+1}v^{2n+2}(1+cs)^{2n+2}},$$
$$y_{24n-8} = (-1)^n\frac{(gt)^{2n}(1+aq)^{2n}}{l^{2n-1}a^{2n}(-1+gq)^n(1+gq)^n(1+2dt)^n},$$
$$y_{24n-7} = (-1)^n\frac{(hu)^{2n}(1+br)^{2n}}{m^{2n-1}b^{2n}(-1+hr)^n(1+hr)^n(1+2eu)^n},$$
$$y_{24n-6} = (-1)^n\frac{(kv)^{2n}(1+cs)^{2n}}{p^{2n-1}c^{2n}(-1+ks)^n(1+ks)^n(1+2fv)^n},$$
$$y_{24n-5} = (-1)^n q\frac{(al)^{2n}(1+dt)^{2n}}{(gt)^{2n}(-1+dl)^n(1+dl)^n(1+2aq)^n},$$

$$y_{24n-4} = (-1)^n r \frac{(bm)^{2n}(1+eu)^{2n}}{(hu)^{2n}(-1+em)^n(1+em)^n(1+2br)^n},$$
$$y_{24n-3} = (-1)^n s \frac{(cp)^{2n}(1+fv)^{2n}}{(kv)^{2n}(-1+fp)^n(1+fp)^n(1+2cs)^n},$$
$$y_{24n-2} = (-1)^n \frac{g^{2n}t^{2n+1}(1+aq)^{2n}}{(al)^{2n}(-1+gq)^n(1+gq)^n(1+2dt)^n},$$
$$y_{24n-1} = (-1)^n \frac{h^{2n}u^{2n+1}(1+br)^{2n}}{(bm)^{2n}(-1+hr)^n(1+hr)^n(1+2eu)^n},$$
$$y_{24n} = (-1)^n \frac{k^{2n}v^{2n+1}(1+cs)^{2n}}{(cp)^{2n}(-1+ks)^n(1+ks)^n(1+2fv)^n},$$
$$y_{24n+1} = (-1)^{n+1} d \frac{a^{2n}l^{2n+1}(1+dt)^{2n}}{t^{2n}g^{2n+1}(-1+dl)^n(1+dl)^{n+1}(1+2aq)^n},$$
$$y_{24n+2} = (-1)^{n+1} e \frac{b^{2n}m^{2n+1}(1+eu)^{2n}}{u^{2n}h^{2n+1}(-1+em)^n(1+em)^{n+1}(1+2br)^n},$$
$$y_{24n+3} = (-1)^{n+1} f \frac{c^{2n}p^{2n+1}(1+fv)^{2n}}{v^{2n}k^{2n+1}(-1+fp)^n(1+fp)^{n+1}(1+2cs)^n},$$
$$y_{24n+4} = (-1)^{n+1} \frac{(gt)^{2n+1}(1+aq)^{2n+1}}{l^{2n}a^{2n+1}(-1+gq)^n(1+gq)^{n+1}(1+2dt)^n},$$
$$y_{24n+5} = (-1)^{n+1} \frac{(hu)^{2n+1}(1+br)^{2n+1}}{m^{2n}b^{2n+1}(-1+hr)^n(1+hr)^{n+1}(1+2eu)^n},$$
$$y_{24n+6} = (-1)^{n+1} \frac{(kv)^{2n+1}(1+cs)^{2n+1}}{p^{2n}c^{2n+1}(-1+ks)^n(1+ks)^{n+1}(1+2fv)^n},$$
$$y_{24n+7} = (-1)^n q \frac{(al)^{2n+1}(1+dt)^{2n+1}}{(gt)^{2n+1}(-1+dl)^n(1+dl)^{n+1}(1+2aq)^{n+1}},$$
$$y_{24n+8} = (-1)^n r \frac{(bm)^{2n+1}(1+eu)^{2n+1}}{(hu)^{2n+1}(-1+em)^n(1+em)^{n+1}(1+2br)^{n+1}},$$
$$y_{24n+9} = (-1)^n s \frac{(cp)^{2n+1}(1+fv)^{2n+1}}{(kv)^{2n+1}(-1+fp)^n(1+fp)^{n+1}(1+2cs)^{n+1}},$$
$$y_{24n+10} = (-1)^{n+1} \frac{g^{2n+1}t^{2n+2}(1+aq)^{2n+1}}{(al)^{2n+1}(-1+gq)^n(1+gq)^{n+1}(1+2dt)^{n+1}},$$
$$y_{24n+11} = (-1)^{n+1} \frac{h^{2n+1}u^{2n+2}(1+br)^{2n+1}}{(bm)^{2n+1}(-1+hr)^n(1+hr)^{n+1}(1+2eu)^{n+1}},$$
$$y_{24n+12} = (-1)^{n+1} \frac{k^{2n+1}v^{2n+2}(1+cs)^{2n+1}}{(cp)^{2n+1}(-1+ks)^n(1+ks)^{n+1}(1+2fv)^{n+1}},$$
$$y_{24n+13} = (-1)^n d \frac{a^{2n+1}l^{2n+2}(1+dt)^{2n+1}}{t^{2n+1}g^{2n+2}(-1+dl)^{n+1}(1+dl)^{n+1}(1+2aq)^{n+1}},$$
$$y_{24n+14} = (-1)^n e \frac{b^{2n+1}m^{2n+2}(1+eu)^{2n+1}}{u^{2n+1}h^{2n+2}(-1+em)^{n+1}(1+em)^{n+1}(1+2br)^{n+1}},$$
$$y_{24n+15} = (-1)^n f \frac{c^{2n+1}p^{2n+2}(1+fv)^{2n+1}}{v^{2n+1}k^{2n+2}(-1+fp)^{n+1}(1+fp)^{n+1}(1+2cs)^{n+1}}.$$

Proof. For $n = 0$ the result holds. Now, suppose that $n > 0$ and that our assumption holds for $n-1$. That is,

$$x_{24n-32} = (-1)^{n-1}\frac{(gt)^{2n-2}(-1+dl)^{n-1}(1+dl)^{n-1}(1+2aq)^{n-1}}{a^{2n-3}l^{2n-2}(1+dt)^{2n-2}},$$
$$x_{24n-31} = (-1)^{n-1}\frac{(hu)^{2n-2}(-1+em)^{n-1}(1+em)^{n-1}(1+2br)^{n-1}}{b^{2n-3}m^{2n-2}(1+eu)^{2n-2}},$$
$$x_{24n-30} = (-1)^{n-1}\frac{(kv)^{2n-2}(-1+fp)^{n-1}(1+fp)^{n-1}(1+2cs)^{n-1}}{c^{2n-3}p^{2n-2}(1+fv)^{2n-2}},$$
$$x_{24n-29} = (-1)^{n-1}d\frac{(al)^{2n-2}(-1+gq)^{n-1}(1+gq)^{n-1}(1+2dt)^{n-1}}{(gt)^{2n-2}(1+aq)^{2n-2}},$$
$$x_{24n-28} = (-1)^{n-1}e\frac{(bm)^{2n-2}(-1+hr)^{n-1}(1+hr)^{n-1}(1+2eu)^{n-1}}{(hu)^{2n-2}(1+br)^{2n-2}},$$
$$x_{24n-27} = (-1)^{n-1}f\frac{(cp)^{2n-2}(-1+ks)^{n-1}(1+ks)^{n-1}(1+2fv)^{n-1}}{(kv)^{2n-2}(1+cs)^{2n-2}},$$
$$x_{24n-26} = (-1)^{n-1}\frac{t^{2n-2}g^{2n-1}(-1+dl)^{n-1}(1+dl)^{n-1}(1+2aq)^{n-1}}{(al)^{2n-2}(1+dt)^{2n-2}},$$
$$x_{24n-25} = (-1)^{n-1}\frac{u^{2n-2}h^{2n-1}(-1+em)^{n-1}(1+em)^{n-1}(1+2br)^{n-1}}{(bm)^{2n-2}(1+eu)^{2n-2}},$$
$$x_{24n-24} = (-1)^{n-1}\frac{v^{2n-2}k^{2n-1}(-1+fp)^{n-1}(1+fp)^{n-1}(1+2cs)^{n-1}}{(cp)^{2n-2}(1+fv)^{2n-2}},$$
$$x_{24n-23} = (-1)^{n-1}q\frac{l^{2n-2}a^{2n-1}(-1+gq)^{n-1}(1+gq)^{n-1}(1+2dt)^{n-1}}{g^{2n-2}t^{2n-1}(1+aq)^{2n-1}},$$
$$x_{24n-22} = (-1)^{n-1}r\frac{m^{2n-2}b^{2n-1}(-1+hr)^{n-1}(-1+hr)^{n-1}(1+2eu)^{n-1}}{h^{2n-2}u^{2n-1}(1+br)^{2n-1}},$$
$$x_{24n-21} = (-1)^{n-1}s\frac{p^{2n-2}c^{2n-1}(-1+ks)^{n-1}(1+ks)^{n-1}(1+2fv)^{n-1}}{k^{2n-2}v^{2n-1}(1+cs)^{2n-1}},$$
$$x_{24n-20} = (-1)^{n}\frac{(gt)^{2n-1}(-1+dl)^{n-1}(1+dl)^{n}(1+2aq)^{n-1}}{a^{2n-2}l^{2n-1}(1+dt)^{2n-1}},$$
$$x_{24n-19} = (-1)^{n}\frac{(hu)^{2n-1}(-1+em)^{n-1}(1+em)^{n}(1+2br)^{n-1}}{b^{2n-2}m^{2n-1}(1+eu)^{2n-1}},$$
$$x_{24n-18} = (-1)^{n}\frac{(kv)^{2n-1}(-1+fp)^{n-1}(1+fp)^{n}(1+2cs)^{n-1}}{c^{2n-2}p^{2n-1}(1+fv)^{2n-1}},$$
$$x_{24n-17} = (-1)^{n-1}d\frac{(al)^{2n-1}(-1+gq)^{n-1}(1+gq)^{n}(1+2dt)^{n-1}}{(gt)^{2n-1}(1+aq)^{2n-1}},$$
$$x_{24n-16} = (-1)^{n-1}e\frac{(bm)^{2n-1}(-1+hr)^{n-1}(1+hr)^{n}(1+2eu)^{n-1}}{(hu)^{2n-1}(1+br)^{2n-1}},$$

$$x_{24n-15} = (-1)^{n-1} f \frac{(cp)^{2n-1}(-1+ks)^{n-1}(1+ks)^{n}(1+2fv)^{n-1}}{(kv)^{2n-1}(1+cs)^{2n-1}},$$
$$x_{24n-14} = (-1)^{n} \frac{t^{2n-1}g^{2n}(-1+dl)^{n-1}(1+dl)^{n}(1+2aq)^{n}}{(al)^{2n-1}(1+dt)^{2n-1}},$$
$$x_{24n-13} = (-1)^{n} \frac{u^{2n-1}h^{2n}(-1+em)^{n-1}(1+em)^{n}(1+2br)^{n}}{(bm)^{2n-1}(1+eu)^{2n-1}},$$
$$x_{24n-12} = (-1)^{n} \frac{v^{2n-1}k^{2n}(-1+fp)^{n-1}(1+fp)^{n}(1+2cs)^{n}}{(cp)^{2n-1}(1+fv)^{2n-1}},$$
$$x_{24n-11} = (-1)^{n} q \frac{l^{2n-1}a^{2n}(-1+gq)^{n-1}(1+gq)^{n}(1+2dt)^{n}}{g^{2n-1}t^{2n}(1+aq)^{2n}},$$
$$x_{24n-10} = (-1)^{n-1} r \frac{m^{2n-1}b^{2n}(-1+hr)^{n-1}(1+hr)^{n}(1+2eu)^{n}}{h^{2n-1}u^{2n}(1+br)^{2n}},$$
$$x_{24n-9} = (-1)^{n} s \frac{p^{2n-1}c^{2n}(-1+ks)^{n-1}(1+ks)^{n}(1+2fv)^{n}}{k^{2n-1}v^{2n}(1+cs)^{2n}},$$
$$y_{24n-32} = (-1)^{n-1} \frac{(gt)^{2n-2}(1+aq)^{2n-2}}{l^{2n-3}a^{2n-2}(-1+gq)^{n-1}(1+gq)^{n-1}(1+2dt)^{n-1}},$$
$$y_{24n-31} = (-1)^{n-1} \frac{(hu)^{2n-2}(1+br)^{2n-2}}{m^{2n-3}b^{2n-2}(-1+hr)^{n-1}(1+hr)^{n-1}(1+2eu)^{n-1}},$$
$$y_{24n-30} = (-1)^{n-1} \frac{(kv)^{2n-2}(1+cs)^{2n-2}}{p^{2n-3}c^{2n-2}(-1+ks)^{n-1}(1+ks)^{n-1}(1+2fv)^{n-1}},$$
$$y_{24n-29} = (-1)^{n-1} q \frac{(al)^{2n-2}(1+dt)^{2n-2}}{(gt)^{2n-2}(-1+dl)^{n-1}(1+dl)^{n-1}(1+2aq)^{n-1}},$$
$$y_{24n-28} = (-1)^{n-1} r \frac{(bm)^{2n-2}(1+eu)^{2n-2}}{(hu)^{2n-2}(-1+em)^{n-1}(1+em)^{n-1}(1+2br)^{n-1}},$$
$$y_{24n-27} = (-1)^{n-1} s \frac{(cp)^{2n-2}(1+fv)^{2n-2}}{(kv)^{2n-2}(-1+fp)^{n-1}(1+fp)^{n-1}(1+2cs)^{n-1}},$$
$$y_{24n-26} = (-1)^{n-1} \frac{g^{2n-2}t^{2n-1}(1+aq)^{2n-2}}{(al)^{2n-2}(-1+gq)^{n-1}(1+gq)^{n-1}(1+2dt)^{n-1}},$$
$$y_{24n-25} = (-1)^{n-1} \frac{h^{2n-2}u^{2n-1}(1+br)^{2n-2}}{(bm)^{2n-2}(-1+hr)^{n-1}(1+hr)^{n-1}(1+2eu)^{n-1}},$$
$$y_{24n-24} = (-1)^{n-1} \frac{k^{2n-2}v^{2n-1}(1+cs)^{2n-2}}{(cp)^{2n-2}(-1+ks)^{n-1}(1+ks)^{n-1}(1+2fv)^{n-1}},$$
$$y_{24n-23} = (-1)^{n} d \frac{a^{2n-2}l^{2n-1}(1+dt)^{2n-2}}{t^{2n-2}g^{2n-1}(-1+dl)^{n-1}(1+dl)^{n}(1+2aq)^{n-1}},$$
$$y_{24n-22} = (-1)^{n} e \frac{b^{2n-2}m^{2n-1}(1+eu)^{2n-2}}{u^{2n-2}h^{2n-1}(-1+em)^{n-1}(1+em)^{n}(1+2br)^{n-1}},$$

$$y_{24n-21} = (-1)^n f \frac{c^{2n-2}p^{2n-1}(1+fv)^{2n-2}}{v^{2n-2}k^{2n-1}(-1+fp)^{n-1}(1+fp)^n(1+2cs)^{n-1}},$$
$$y_{24n-20} = (-1)^n \frac{(gt)^{2n-1}(1+aq)^{2n-1}}{l^{2n-2}a^{2n-1}(-1+gq)^{n-1}(1+gq)^n(1+2dt)^{n-1}},$$
$$y_{24n-19} = (-1)^n \frac{(hu)^{2n-1}(1+br)^{2n-1}}{m^{2n-2}b^{2n-1}(-1+hr)^{n-1}(1+hr)^n(1+2eu)^{n-1}},$$
$$y_{24n-18} = (-1)^n \frac{(kv)^{2n-1}(1+cs)^{2n-1}}{p^{2n-2}c^{2n-1}(-1+ks)^{n-1}(1+ks)^n(1+2fv)^{n-1}},$$
$$y_{24n-17} = (-1)^{n-1} q \frac{(al)^{2n-1}(1+dt)^{2n-1}}{(gt)^{2n-1}(-1+dl)^{n-1}(1+dl)^n(1+2aq)^n},$$
$$y_{24n-16} = (-1)^{n-1} r \frac{(bm)^{2n-1}(1+eu)^{2n-1}}{(hu)^{2n-1}(-1+em)^{n-1}(1+em)^n(1+2br)^n},$$
$$y_{24n-15} = (-1)^{n-1} s \frac{(cp)^{2n-1}(1+fv)^{2n-1}}{(kv)^{2n-1}(-1+fp)^{n-1}(1+fp)^n(1+2cs)^n},$$
$$y_{24n-14} = (-1)^n \frac{g^{2n-1}t^{2n}(1+aq)^{2n-1}}{(al)^{2n-1}(-1+gq)^{n-1}(1+gq)^n(1+2dt)^n},$$
$$y_{24n-13} = (-1)^n \frac{h^{2n-1}u^{2n}(1+br)^{2n-1}}{(bm)^{2n-1}(-1+hr)^{n-1}(1+hr)^n(1+2eu)^n},$$
$$y_{24n-12} = (-1)^n \frac{k^{2n-1}v^{2n}(1+cs)^{2n-1}}{(cp)^{2n-1}(-1+ks)^{n-1}(1+ks)^n(1+2fv)^n},$$
$$y_{24n-11} = (-1)^{n-1} d \frac{a^{2n-1}l^{2n}(1+dt)^{2n-1}}{t^{2n-1}g^{2n}(-1+dl)^n(1+dl)^n(1+2aq)^n},$$
$$y_{24n-10} = (-1)^{n-1} e \frac{b^{2n-1}m^{2n}(1+eu)^{2n-1}}{u^{2n-1}h^{2n}(-1+em)^n(1+em)^n(1+2br)^n},$$
$$y_{24n-9} = (-1)^{n-1} f \frac{c^{2n-1}p^{2n}(1+fv)^{2n-1}}{v^{2n-1}k^{2n}(-1+fp)^n(1+fp)^n(1+2cs)^n}.$$

From system (4), we get

$$x_{24n-8} = \frac{y_{24n-14}x_{24n-17}}{y_{24n-11}\left(1+y_{24n-14}x_{24n-17}\right)}$$
$$= \frac{\left(\begin{array}{c} (-1)^n \frac{g^{2n-1}t^{2n}(1+aq)^{2n-1}}{(al)^{2n-1}(-1+gq)^{n-1}(1+gq)^n(1+2dt)^n} \\ \times(-1)^{n-1} d \frac{(al)^{2n-1}(-1+gq)^{n-1}(1+gq)^n(1+2dt)^{n-1}}{(gt)^{2n-1}(1+aq)^{2n-1}} \end{array}\right)}{\begin{array}{c} (-1)^{n-1} d \frac{a^{2n-1}l^{2n}(1+dt)^{2n-1}}{t^{2n-1}g^{2n}(-1+dl)^n(1+dl)^n(1+2aq)^n} \\ \left(1+\left(\begin{array}{c} (-1)^n \frac{g^{2n-1}t^{2n}(1+aq)^{2n-1}}{(al)^{2n-1}(-1+gq)^{n-1}(1+gq)^n(1+2dt)^n} \\ \times(-1)^{n-1} d \frac{(al)^{2n-1}(-1+gq)^{n-1}(1+gq)^n(1+2dt)^{n-1}}{(gt)^{2n-1}(1+aq)^{2n-1}} \end{array}\right)\right) \end{array}}$$

$$= \frac{-dt/(1+2dt)}{(-1)^{n-1} d \frac{a^{2n-1} l^{2n} (1+dt)^{2n-1}}{t^{2n-1} g^{2n} (-1+dl)^n (1+dl)^n (1+2aq)^n} \left(1 - \frac{dt}{(1+2dt)}\right)}$$
$$= \frac{-t^{2n} g^{2n} (-1+dl)^n (1+dl)^n (1+2aq)^n}{(-1)^{n-1} a^{2n-1} l^{2n} (1+dt)^{2n-1} (1+dt)}$$

Hence,

$$x_{24n-8} = (-1)^n \frac{(gt)^{2n} (-1+dl)^n (1+dl)^n (1+2aq)^n}{a^{2n-1} l^{2n} (1+dt)^{2n}}.$$

Similarly, we can prove the other relations.

2.5 Numerical Examples

In order to confirm our theoretical results, we consider in this section some numerical examples.

Example 1. Consider the system (1) with the initial conditions $x_{-8} = -7$, $x_{-7} = 0.2$, $x_{-6} = 0.4$, $x_{-5} = 6$, $x_{-4} = 12$, $x_{-3} = 2.6$, $x_{-2} = 0.6$, $x_{-1} = 0.2$, $x_0 = 1.2$, $y_{-8} = 5$, $y_{-7} = 2.5$, $y_{-6} = 1.5$, $y_{-5} = 2.2$, $y_{-4} = 1.5$, $y_{-3} = 11$, $y_{-2} = 2$, $y_{-1} = 0.5$, and $y_0 = -4$. See Figure 1.

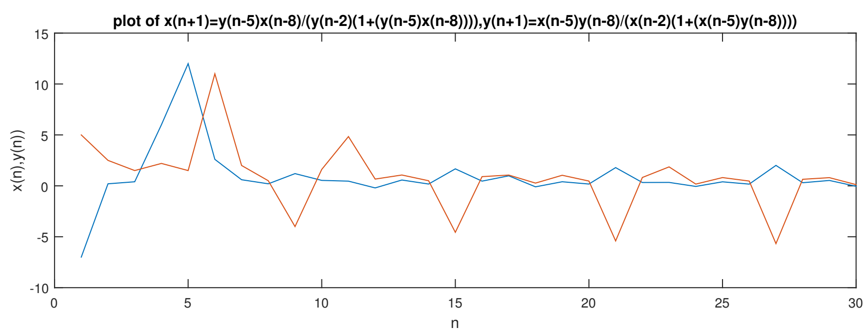

Figure 1.

Example 2. Figure 2. below describe the solutions of system (2) when $x_{-8} = 0.7$, $x_{-7} = 11$, $x_{-6} = -0.3$, $x_{-5} = -9$, $x_{-4} = 7$, $x_{-3} = -6.2$, $x_{-2} = 5$, $x_{-1} = 0.3$, $x_0 = 2$, $y_{-8} = -2$, $y_{-7} = 9$, $y_{-6} = 2.5$, $y_{-5} = -9$, $y_{-4} = 0.3$, $y_{-3} = 7$, $y_{-2} = 2.2$, $y_{-1} = 14$ and $y_0 = -0.3$.

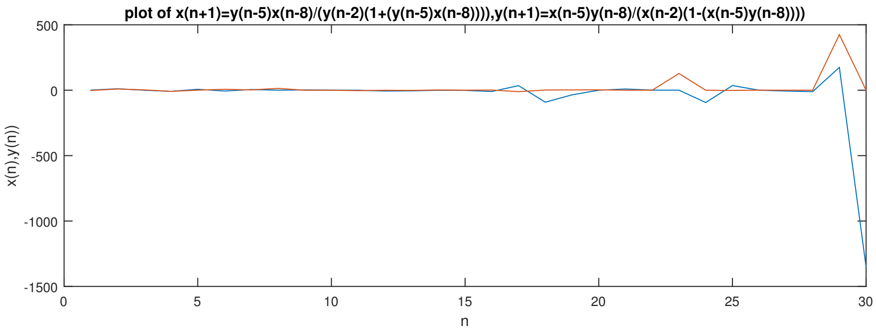

Figure 2.

Example 3. See Figure 3, when we take system (3) and put $x_{-8} = 1.8$, $x_{-7} = 5$, $x_{-6} = 0.9$, $x_{-5} = 0.2$, $x_{-4} = -1.5$, $x_{-3} = 2.5$, $x_{-2} = 0.6$, $x_{-1} = 1.3$, $x_0 = 4$, $y_{-8} = 1.6$, $y_{-7} = 0.3$, $y_{-6} = 1.2$, $y_{-5} = 1.3$, $y_{-4} = 0.9$, $y_{-3} = -1.5$, $y_{-2} = 6.2$, $y_{-1} = 1.6$ and $y_0 = 4.3$.

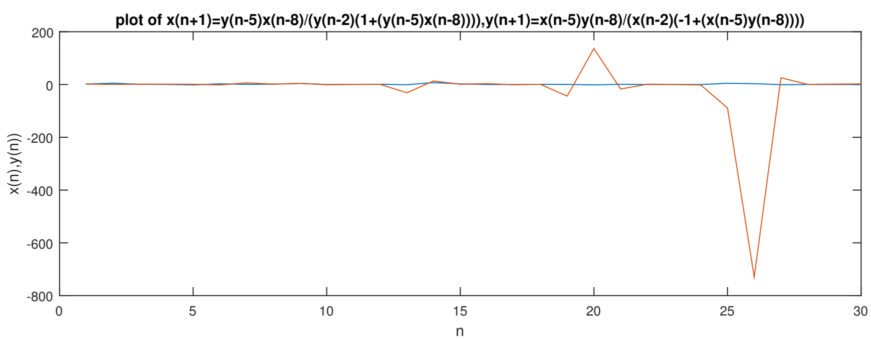

Figure 3.

Example 4. Consider the system (4) when $x_{-8} = 2.1$, $x_{-7} = 0.6$, $x_{-6} = 1.8$, $x_{-5} = -1.5$, $x_{-4} = 10.2$, $x_{-3} = -11$, $x_{-2} = 3$, $x_{-1} = 6.3$, $x_0 = 11.4$, $y_{-8} = 3.3$, $y_{-7} = -1.5$, $y_{-6} = 0.9$, $y_{-5} = 12.4$, $y_{-4} = 3.2$, $y_{-3} = 12$, $y_{-2} = 2$, $y_{-1} = 6.6$ and $y_0 = 1.5$. See Figure 4.

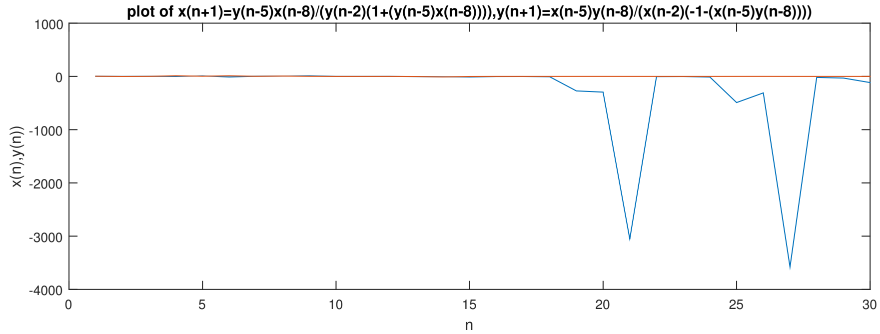

Figure 4.

References

[1] A. M. Ahmed and E. M. Elsayed, The Expressions of Solutions and The Periodicity of Some Rational Difference Equations System, J. Appl. Math. Informatics, 34 (1-2) (2016), 35-48.

[2] A. M. Alotaibi, M. S. M. Noorani and M. A. El-Moneam, On the asymptotic behavior of some nonlinear difference equations, Journal of Computational Analysis and Applications, 26 (2019), 604–627.

[3] N. Battaloglu, C. Cinar and I. Yalcinkaya, The dynamics of the difference equation $x_{n+1} = (\alpha x_{n-m})/(\beta + \gamma x^{p}_{n-(k+1)})$, Ars Combinatoria, 97 (2010), 281–288.

[4] I. Dekkar, N. Touafek and Y. Yazlik, Global stability of a third-order nonlinear system of difference equations with period-two coefficients, Revista de la Real Academia de Ciencias Exactas, Fsicas y Naturales. Serie A. Matemticas, 111 (2) (2017), 325–347.

[5] D. S. Dilip and S. M. Mathew, Dynamics of a second order nonlinear difference system with exponents, Journal of the Egyptian Mathematical Society, (2021) 29:10.

[6] D. S .Dilip, S. M. Mathew and E. M. Elsayed, Asymptotic and boundedness behaviour of a second order difference equation, Journal of Computational Mathematica,4 (2) (2020), 68-77.

[7] Q. Din, Qualitative nature of a discrete predator-prey system, Contem. Methods Math. Phys. Grav., 1 (2015), 27-42.

[8] M. M. El-Dessoky, The form of solutions and periodicity for some systems of third-order rational difference equations, Math. Meth. Appl. Sci., 39 (2016), 1076–1092.

[9] M. M. El-Dessoky, Solution for rational systems of difference equations of order three, Mathematics, 4 (3) (2016), 1-12.

[10] M. M. El-Dessoky and E. M. Elsayed, On the solutions and periodic nature of some systems of rational difference equations, J. Computational Analysis and Applications, 18 (2) (2015), 206-218.

[11] M. M. El-Dessoky, A. Khaliq and A. Asiri, On some rational system of difference equations, J. Nonlinear Sci. Appl., 11 (2018), 49-72.

[12] E. M. Elsayed , Qualitative behavior of a rational recursive sequence, Indag. Math.,19 (2) (2008), 89-201.

[13] E. M. Elsayed, Solution and attractivity for a rational recursive sequence, Discrete Dynamics in Nature and Society, Vol. 2011 (2011), Article ID 982309, 17 pages.

[14] E. M. Elsayed, F. Alzahrani, I. Abbas and N. H. Alotaibi, Dynamical behavior and solution of nonlinear difference equation via Fibonacci sequence, Journal of Applied Analysis & Computation, 10 (2020), 281-288.

[15] E. M. Elsayed, A solution form of a class of rational difference equations, International Journal of Nonlinear Science, 8(4) (2009), 402-411.

[16] E. M. Elsayed, Dynamics of a rational recursive sequence, International Journal of Difference Equations, 4(2)(2009), 185–200.

[17] E. M. Elsayed, Expressions of solution for a class of difference equations, Analele Stiintifice ale Universitatii Ovidius Constanta, 18(2) (2010), 99-114.

[18] E. M. Elsayed, B. S. Alofi, and A. Q. Khan, Solution Expressions of Discrete Systems of Difference Equations, Mathematical Problems in Engineering, Volume 2022 (2022), Article ID 3678257, 14 pages.

[19] E. M. Elsayed, Q. Din and N. A. Bukhary, Theoretical and numerical analysis of solutions of some systems of nonlinear difference equations, AIMS Mathematics, 7(8) (2022), 15532–15549.

[20] E. M. Elsayed, Solutions of rational difference system of order two, Math. Comput. Mod., 55 (2012), 378–384.

[21] E. M. Elsayed and K. N. Alharbi, The expressions and behavior of solutions for nonlinear systems of rational difference equations, Journal of Innovative Applied Mathematics and Computational Sciences (JIAMCS), 2 (1) (2022), 78–91.

[22] E. M. Elsayed and T. F. Ibrahim, Periodicity and solutions for some systems of nonlinear rational difference equations, Hacettepe Journal of Mathematics and Statistics, 44 (6) (2015), 1361-1390.

[23] E. M. Elsayed and A. Alghamdi, The form of the solutions of nonlinear difference equations systems, J. Nonlinear Sci. Appl., 9 (2016), 3179-3196.

[24] E. M. Elsayed and A. Alshareef, Qualitative behavior of a system of second order difference equations, European Journal of Mathematics and Applications, 1:15 (2021), 1-11.

[25] E. M. Elsayed and H. S. Gafel, Some systems of three nonlinear difference equations, Journal of Computational Analysis and Applications, 29 (1) (2021), 86-108.

[26] E. M. Elsayed and N. H. Alotaibi, The form of the solutions and behavior of some systems of nonlinear difference equations, Dynamics of Continuous, Discrete and Impulsive Systems, Series A: Mathematical Analysis, 27 (2020), 283-297.

[27] E. M. Elsayed and A. Alghamdi, Qualitative behavior of a rational recursive sequence of second order, Rocky Mountain Journal of Mathematics, 49 (7) (2019), 2135-2154.

[28] E. M. Elsayed and B. Alofi, Stability analysis and periodicitly properties of a class of rational difference equations, MANAS Journal of Engineering, 10 (2) (2022), 203-210.

[29] E. M. Elsayed and J. G. AL-Juaid, The form of solutions and periodic nature for some system of difference equations, Fundamental Journal of Mathematics and Applications, 6 (1) (2023), 24-34.

[30] E. M. Elsayed and F. Alzahrani, Periodicity and solutions of some rational difference equations systems, Journal of Applied Analysis and Computation, 9 (6) (2019), 2358-2380.

[31] M. Gumus and O. Ocalan, The qualitative analysis of a rational system of difference equations, Journal of Fractional Calculus and Applications, 9 (2) (2018), 113-126.

[32] N. Haddad, N. Touafek and E. M. Elsayed, A note on a system of difference equations, Analele Stiintifice ale universitatii al i cuza din iasi serie noua matematica, LXIII (3) (2017), 599-606.

[33] N. Haddad , N. Touafek and J. F. T. Rabago, Solution form of a higher-order system of difference equations and dynamical behavior of its special case, Math Meth. Appl. Sci., 40 (2017), 3599–3607.

[34] N. Haddad, N. Touafek and J. T. Rabago, Well-defined solutions of a system of difference equations, J. Appl. Math. Comput., 56 (1-2) (2018), 439–458.

[35] Y. Halim, A system of difference equations with solutions associated to Fibonacci numbers, International Journal of Difference Equations, 11 (1) (2016), 65–77.

[36] T. F. Ibrahim, Asymptotic behavior of a difference equation model in exponential form, Mathematical Methods in the Applied Sciences, 45 (17) (2022), 10736-10748.

[37] G. Hu, Global Behavior of A System of Two Nonlinear Difference Equation, World Journal of Research and Review (WJRR), 2 (6) (2016), 36-38.

[38] M. Kara and Y. Yazlık, On a solvable system of non-linear difference equations with variable coefficients, Journal of Science and Arts,1(54) (2021), 145-162.

[39] M. Kara and Y. Yazlik, On the solutions of three-dimensional system of difference equations via recursive relations of order two and applications, Journal of Applied Analysis and Computation, 12 (2) (2022), 736–753.

[40] A. Khaliq, M. Zubair and A. Q. Khan, Asymptotic Behavior of the Solutions of Difference Equation System of Exponential Form, Fractals, 28 (6) (2020), 118-128, doi: 10.1142/S0218348X20501182.

[41] A. Khaliq and M. Shoaib, Dynamics of three-dimensional system of second order rational difference equations, Electronic Journal of Mathematical Analysis and Applications, 9(2) (2021), 308-319.

[42] A. Khaliq and E. M. Elsayed, The dynamics and solution of some difference equations, J. Nonlinear Sci. Appl., 9 (2016), 1052-1063.

[43] A. Khaliq, T. F. Ibrahim, A. M. Alotaibi, M. Shoaib and M. El-Moneam, Dynamical analysis of discrete-time two-predators one-prey Lotka–Volterra model, Mathematics, 2022 (10) (2022), 4015.

[44] A. S. Kurbanli, On the behavior of solutions of the system of rational difference equations $x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1)$, $y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1)$, World Applied Sciences Journal, 10 (11) (2010), 1344-1350.

[45] A. S. Kurbanli, C. Çinar and D. Şimşek, On the periodicity of solutions of the system of rational difference equations $x_{n+1} = x_{n-1} + y_n/(y_n x_{n-1} - 1)$, $y_{n+1} = y_{n-1} + x_n/(x_n y_{n-1} - 1)$, Applied Mathematics, 2 (2011), 410-413.

[46] A. S. Kurbanli, C. Çinar and I. Yalçinkaya,On the behavior of positive solutions of the system of rational difference equations $x_{n+1} = x_{n-1}/(y_n x_{n-1} + 1)$, $y_{n+1} = y_{n-1}/(x_n y_{n-1} + 1)$, Mathematical and Computer Modelling, 53 (2011), 1261–1267.

[47] M. Mansour, M. M. El-Dessoky and E. M. Elsayed, The form of the solutions and periodicity of some systems of difference equations, Discrete Dynamics in Nature and Society, 2012 (2012), ID 406821, 1-17.

[48] B. Ogul and D. Simsek, On the recursive sequence, Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications & Algorithms, 29 (2022), 423-435.

[49] A. Sanbo and E. M. Elsayed, Analytical Study of a System of Difference Equation, Asian Research Journal of Mathematics, 14 (1) (2019), 1-18.

[50] L. Sh. Aljoufi and M. B. Almatrafi, Dynamical analysis and solutions of nonlinear difference equations of twenty-fourth order, New Trends in Mathematical Sciences, 10 (4) (2022), 80-92.

[51] N. Touafek and E. M. Elsayed, On the solutions of systems of rational difference equations, Math. Comput. Mod. , 55 (2012), 1987-1997.

[52] D. Tollu and I. Yalcınkaya, On solvability of a three-dimensional system of nonlinear difference equations, Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications & Algorithms, 29 (2022), 35-47.

[53] R. Vivek, E. M. Elsayed, K. Kanagarajan, D. Vivek, Qualitative analysis of quaternion fuzzy fractional differential equations with Ξ-Hilfer fractional derivative, Pure and Applicable Analysis, 2022 (2022) , No. 6, 1-16

[54] J. L. Williams, On a class of nonlinear max-type difference equations, Cogent Mathematics,3 (2016), 1269597, 1-11.

[55] Y. Yazlik, D. T. Tollu and N. Taskara, On the solutions of a max-type difference equation system, Math. Meth. Appl. Sci., 38 (2015), 4388–4410.

[56] Q. Zhang, J. Liu and Z. Luo, Dynamical Behavior of a System of Third-Order Rational Difference Equation, Discrete Dynamics in Nature and Society, Vol. 2015 (2015), Article ID 530453, 6 pages.